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Variable Ring Light Imaging: Capturing Transient Subsurface Scattering with An Ordinary Camera Supplementary Material

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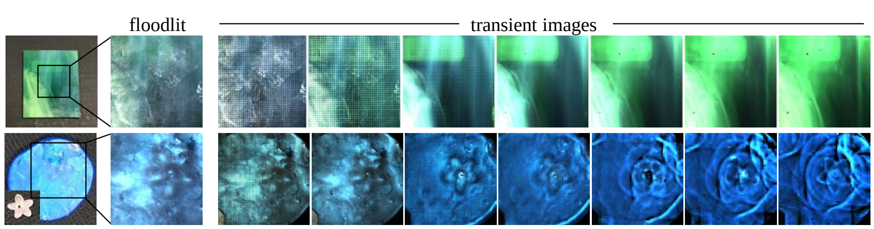
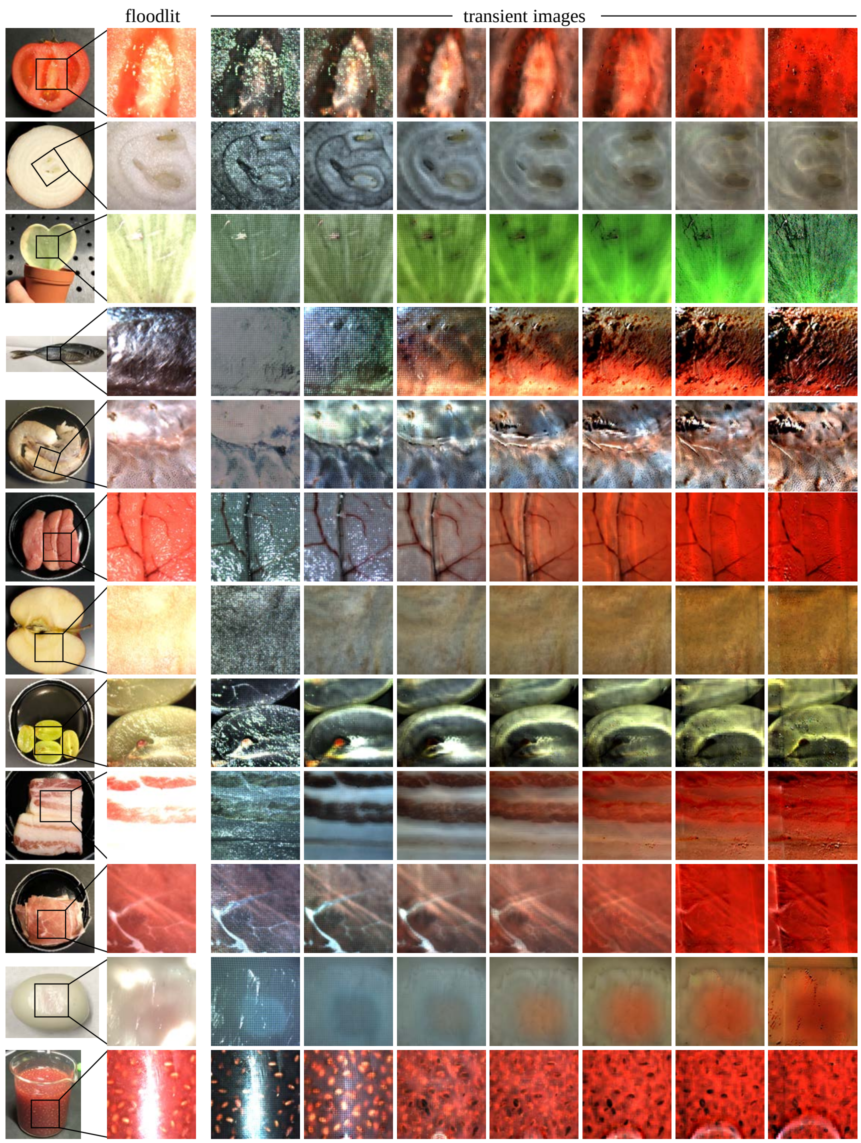
A supplemental movie is also available.

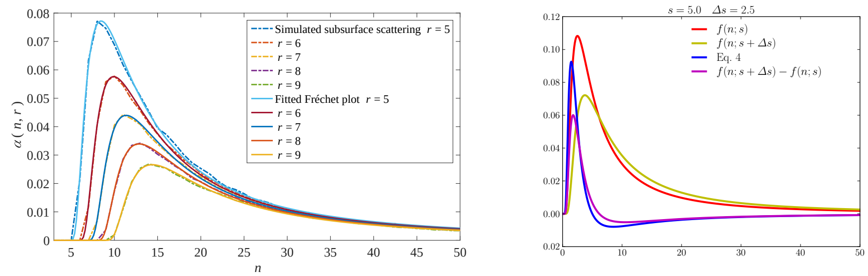
1 Additional Experimental Results

The recovered transient images reveal the intrinsic structures of the surface as each of their pixels encodes integrated interactions of light as it propagates in the subsurface volume. The volumetric composition of the subsurface and their colors that are otherwise unrecognizable from the outer surface become apparent in the transient images. Note that the first transient image encodes the surface reflection which includes specular reflection that is often saturated. The larger the ring light radius and the longer the path length, the deeper the color and the wider the spatial propagation of light. The transient light that enters the images from their boundaries (e.g., the bright half-ring from the bottom in the last example of pomegranate juice) are those due to reflection at object boundaries. The transient images of the cactus ends with high frequency spatial patterns due to light exiting from its back surface. As these results demonstrate, variable ring light imaging provides an effective visual tool for probing the volumetric and radiometric structures of real-world surfaces including those of natural objects (e.g., fish, leaf, and fish roe).





floodlit
transient images



(a) (b)

Fig. 1: (a) A representative result of Monte Carlo simulation of subsurface scattering shows that the distribution $\alpha(n, r)$, the ratio of n-bounce light with an r-ring light, has non-zero values sharply rising after a certain n and quickly falling off with overlapping tails for different r. These $\alpha(n, r)$ distributions can be approximated accurately with Fréchet distributions of unit shape parameter and different scale parameters proportional to r. (b) The derivative approximation of the difference of two Fréchet distributions shows that we can safely assume that the difference of two ring light observations of different radii indeed encodes n-bounce light.

2 Transient Images are Approximate n-bounce Images

Fig. 1a shows an example result from a number of Monte Carlo simulations of subsurface scattering with various parameter values plotted as the ratio of n-bounce light for various radii r denoted as $\alpha(n, r)$. We conducted simulations for a number of combinations of different values for g of the Henyey-Greenstein between 0.1 and 0.9 and the light path length with a Gaussian distribution of variance between 0.1 and 0.3 of the mean. The plot shows two important characteristics of n-bounce light. First, we can confirm that $\alpha(n, r)$ takes on values only after an n proportional to r. This empirically shows that n-bounce light observed with an r-ring light is indeed bounded by $n(r)$. Second, we can see that the distribution of $\alpha(n, r)$ across n for various radii r falls off quickly and significantly overlaps with an adjacent $\alpha(n, r + \Delta)$. This sharp rise in front of $n(r)$ is exactly the property we would like to see: the number of bounces of light for a given path length is tightly centered around the mean and thus the ring light image captures light with a well-defined lower bound and the longer path length or larger number of bounces are almost unbounded.

We also notice that the shape of α is unique and resembles a Fréchet distribution, in particular with a unit shape parameter

$$\alpha(n, r) = \beta(r) f(n; s(r)) = \beta(r) \frac{s(r)}{n^2} \exp\left[-\frac{s(r)}{n}\right], \tag{1}$$

where $\beta(r)$ is a scalar that differs for each observation of n-bounce lights for an r-ring light, and $s(r)$ is the scale parameter of the Fréchet distribution $f(n; s(r))$

which varies depending on r. We confirm this conjecture by actually fitting Fréchet distributions to each of $\alpha(n, r)$ for $r = 5, \cdots, 25$ and $n = 1, \cdots, 50$ for 12 different simulations results and finding the relative RMS to be 0.14.

Let us consider two ratios of n-bounce light that are observed for r and $r + \Delta r$-ring lights and denote $s(r) = s$, and $s(r + \Delta r) = s + \Delta s$. As each can be modeled with Fréchet distributions their difference is

$$\begin{aligned} \alpha(n, r) - \alpha(n, r + \Delta r) &= \beta(r) f(n; s) - \beta(r + \Delta r) f(n; s + \Delta s)) \\ &\approx \beta(r) \left(f(n; s) - f(n; s + \Delta s) \right)) , \end{aligned} \tag{2}$$

where, for a sufficiently small Δr, we have assumed $\beta(r) \approx \beta(r + \Delta r)$. If we approximate the derivative of the Fréchet distribution with respect to its shape parameter with forward differentiation, we have

$$f(n; s) - f(n; s + \Delta s) \approx -\Delta s \frac{\partial f(n; s)}{\partial s} = -\Delta s \frac{n - s}{ns} f(n; s) . \tag{3}$$

From Eqs. 2 and 3, we have

$$\alpha(n, r) - \alpha(n, r + \Delta r) \approx -\beta(r) \Delta s \frac{n - s}{ns} f(n; s) . \tag{4}$$

Fig. 1b shows a pair of Fréchet distributions with different shape parameters (s and $s + \Delta s$), their actual differences ($f(n; s) - f(n; s + \Delta s)$), and the approximation using the derivative of the Fréchet distribution with respect to its shape parameter ($\Delta s \frac{n-s}{ns} f(n; s)$), where Δs is about 50% of s. We can see that the approximation (Eq. 4) reflects the shape of the actual difference well. Most important, the derivative sharply rises around $n(s)$ and quickly tapers off. We can thus safely assume that, for mean bounces larger than $n(r + \Delta r)$,

$$\alpha(n, r) \approx \alpha(n, r + \Delta r) . \tag{5}$$

Since the difference of ring light subsurface scattering for two radii captures the $n(r + \Delta r) > n \geq n(r)$-bounces of light

$$E(r) - E(r + \Delta r) \approx \sum_{n(r+\Delta r) > n \geq n(r),\, |\Pi(\ell)| = r} \alpha(n, r) L(n, |\ell|) , \tag{6}$$

where $L(n, |\ell|)$ is the radiance of light whose path length is $|\ell|$ that has experienced n-bounces. Note that we are reusing the notation $L(|\ell|)$ initially used for radiance of a light ray with path length $|\ell|$. In other words, the transient images computed from variable ring light imaging can be considered to encode approximate n-bounce light.